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Revisiting lot sizing for an inventory system with product recovery

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ABSTRACT

This note investigates the study by Teunter (2004) [1] on lot sizing for inventory systems with product recovery where lot sizing formulae for two recovery policies ($(1, R)$ and $(P, 1)$) are derived. Instead of applying the classical optimization technique, we develop an integrated solution procedure for each of the two policies using algebraic approaches. Numerical analysis show that our examples result in a lower total cost for both policies.

1. Introduction

Teunter [1] developed two lot sizing models with product recovery. In the first model, one production lot is alternated with R recovery lots, or $(1, R)$ policy. For the other model, P production lot is alternated with one recovery lot, or $(P, 1)$ policy. He derived two integrated total production inventory costs and three decision variables. They are the optimal production lot size (Q^*_p), the optimal recovery lot size (Q^*_r) and the number of recovery (R) or the production lots (P). The values of Q^*_p and Q^*_r are solved using partial differential equations. Corresponding to Q^*_p and Q^*_r, the values of R or P are then derived. Since R and P must be discrete, the author modified Q^*_p and Q^*_r, so that R or P was discrete. In this paper, we suggest an integrated solution procedure to solve Q^*_p and Q^*_r using a simple algebraic method without derivative. This method is simple and it is helpful for students who are not familiar with calculus.

There has been some research on solving an optimal solution without derivative and three methods are used widely. The methods are algebraic approach, cost-difference comparison method and arithmetic–geometric mean inequality. Grubbstorm [2] was the first to show that a standard economic order quantity model could be solved using an algebraic approach or without using derivative. Grubbstorm and Erdem [3] extended the approach by allowing backorder and Cardenas-Barron [4] applied the algebraic approach to solve the classical economic production quantity (EPQ) model with shortage. Yang and Wee [5] developed an integrated vendor–buyer inventory system derived without derivatives. Wee et al. [6] developed an EOQ model with temporary sale price derived without derivatives. Other researchers who used the algebraic approaches are Chang et al. [7] who solved EOQ and EPQ model with shortage, Sphicas [8] who solved EOQ and EPQ with linear and fixed backorder cost, Wee and Chung [9] who solved the economic lot size for an integrated vendor–buyer inventory system, Cardenas-Barron [10] who used the approach to solve an N-stage-multi-customer supply chain model and Cardenas-Barron [11] who solved inventory policies of immediate rework process model and N-cycle rework process model. Chung and Wee [12] developed an optimal economic lot size for a three-stage supply chain with backordering derived without derivatives. Cost-difference comparison method was introduced by Minner [13] and Wee et al. [14] extended the method by simplifying the solution procedure. Teng [15] was among the first researchers to derive

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Notations

d	demand rate (unit per time)
f	return fraction (unit per time)
p	production rate (unit per time)
r	recovery rate (unit per time)
K_p	ordering (setup) cost per production lot ($ per setup)
K_r	ordering (setup) cost per recovery lot ($ per setup)
h_r	holding cost per recoverable item per time unit ($ per unit per time)
h_s	holding cost per serviceable item per time unit ($ per unit per time)
Q_p	production lot size (unit)
Q_r	recovery lot size (unit)

Assumptions:

1. The return rate is equal to fd where $0 < f < 1$.
2. Production rate and recovery rate are larger than demand rate.
3. All return items are recovered.

EOQ using arithmetic–geometric mean inequality. Cardenas-Barron [16] extended the method and solved EOQ and EPQ model with backorder and Cardenas-Barron [17] presented a discussion on the use of arithmetic–geometric mean method.

2. Optimal solution without derivative

This section shows how the two models using algebraic approaches are solved. To compare our results with Teunter, we use the same notation and assumptions as [1].

2.1. Model 1: $(1, R)$ policy

In the $(1, R)$ policy, manufacturer has one production setup and R rework setups per cycle. The following cost expression of the total inventory cost per unit time is obtained from Eq. (1) of [1]:

$$TC^{(1,R)}(Q_p, Q_r) = \frac{K_p d(1-f)}{Q_p} + \frac{K_r df}{Q_r} + \frac{h_s}{2}\left((1-f)\left(1-\frac{d}{p}\right)Q_p + f\left(1-\frac{d}{r}\right)Q_r\right) + \frac{h_r f}{2}\left(\left(1-\frac{d}{r}\right)Q_r + Q_p\right). \tag{1}$$

From Eq. (2) of [1],

$$RQ_r(1-f) = Q_p f. \tag{2}$$

After rearranging, one has:

$$Q_r = \frac{Q_p f}{R(1-f)}. \tag{3}$$

Teunter [1] differentiated (1) with respect to $Q^*{}_p$ and $Q^*{}_r$ and equating the result to zero, such that:

$$Q_p^{(1,R)} = \sqrt{\frac{2K_p d(1-f)}{h_s(1-f)\left(1-\frac{d}{p}\right)Q_p + h_r f}} \quad \text{and} \quad Q_r^{(1,R)} = \sqrt{\frac{2K_r d}{(h_s+h_r)\left(1-\frac{d}{r}\right)}}. \tag{4}$$

Since R has to be discrete, Teunter modify the optimal R so that the variable is discrete. The modification formulae for $(1, R)$ model is:

$$\tilde{Q_p}^{(1,R)} = \frac{\tilde{R}^{(1,R)} Q_r^{(1,R)}(1-f)}{f} \tag{5}$$

where

$$\tilde{R}^{(1,R)} = \max\{1, [R^{(1,R)}]\} \tag{6}$$

is the positive integer nearest to $R^{(1,R)}$.

For our analysis, we substitute (3) into (1), resulting in:

$$TC^{(1,R)}(Q_p) = \frac{K_p d(1-f)}{Q_p} + \frac{K_r dR(1-f)}{Q_p} + \frac{h_s}{2}\left((1-f)\left(1-\frac{d}{p}\right)Q_p + f^2\left(1-\frac{d}{r}\right)\left(\frac{Q_p}{R(1-f)}\right)\right) + \frac{h_r f}{2}\left(\left(1-\frac{d}{r}\right)\left(\frac{Q_p f}{R(1-f)}\right) + Q_p\right). \quad (7)$$

Eq. (7) can be rewritten as:

$$TC^{(1,R)}(Q_p) = \frac{A_1}{Q_p} + B_1 Q_p \quad (8)$$

where:

$$A_1 = K_p d(1-f) + K_r dR(1-f)$$
$$B_1 = \frac{h_s}{2}\left((1-f)\left(1-\frac{d}{p}\right) + f^2\left(1-\frac{d}{r}\right)\left(\frac{1}{R(1-f)}\right)\right) + \frac{h_r f}{2}\left(\left(1-\frac{d}{r}\right)\left(\frac{f}{R(1-f)}\right) + 1\right).$$

Our objective is to derive the optimal total cost. Rewrite (8) with the method of perfect square format results in:

$$TC^{(1,R)}(Q_p) = \frac{1}{Q_p}\left(\sqrt{A_1} - Q_p\sqrt{B_1}\right)^2 + 2\sqrt{A_1 B_1}. \quad (9)$$

From (9), the minimum TC occurs when $(\sqrt{A_1} - Q_p\sqrt{B_1}) = 0$. Therefore, one has:

$$Q^*{}_p{}^{(1,R)} = \sqrt{\frac{A_1}{B_1}} \quad (10)$$

and

$$TC^{(1,R)}(Q^*{}_p) = 2\sqrt{A_1 B_1}. \quad (11)$$

Substitute the original values of A_1 and B_1 into (10), the optimal production quantity can be modeled as:

$$Q^*{}_p{}^{(1,R)} = \sqrt{\frac{2(K_p d(1-f) + K_r dR(1-f))}{h_s\left((1-f)\left(1-\frac{d}{p}\right) + f^2\left(1-\frac{d}{r}\right)\left(\frac{1}{R(1-f)}\right)\right) + h_r f\left(\left(1-\frac{d}{r}\right)\left(\frac{f}{R(1-f)}\right) + 1\right)}} \quad (12)$$

and Eq. (11) can be rewritten as:

$$TC^{(1,R)}(Q^*{}_p) = 2\left(\frac{X_1}{R} + Y_1 R + K_p d(1-f)\left(\frac{h_r f}{2} + \frac{h_s}{2}(1-f)\left(1-\frac{d}{p}\right)\right) + K_r d f^2\left(\frac{h_r}{2}\left(1-\frac{d}{r}\right) + \frac{h_s}{2}\left(1-\frac{d}{r}\right)\right)\right)^{1/2} \quad (13)$$

where:

$$X_1 = K_p d f^2\left(\frac{h_r}{2}\left(1-\frac{d}{r}\right) + \frac{h_s}{2}\left(1-\frac{d}{r}\right)\right)$$

and

$$Y_1 = K_r d(1-f)\left(\frac{h_r f}{2} + \frac{h_s}{2}(1-f)\left(1-\frac{d}{p}\right)\right).$$

Rewriting (13) using the method of perfect square format results in:

$$TC^{(1,R)}(Q_p) = 2\left(\frac{1}{R}\left(\sqrt{X_1} - R\sqrt{Y_1}\right)^2 + 2\sqrt{X_1 Y_1} + K_p d(1-f)\left(\frac{h_r f}{2} + \frac{h_s}{2}(1-f)\left(1-\frac{d}{p}\right)\right) + K_r d f^2\left(\frac{h_r}{2}\left(1-\frac{d}{r}\right) + \frac{h_s}{2}\left(1-\frac{d}{r}\right)\right)\right)^{1/2}. \quad (14)$$

From (14), TC is minimum when $(\sqrt{X_1} - R\sqrt{Y_1}) = 0$. Therefore, one has:

$$R^{*(1,R)} = \sqrt{\frac{X_1}{Y_1}}. \quad (15)$$

Substitute the original X_1 and Y_1 values into (15), such that:

$$R^{*(1,R)} = \sqrt{\frac{K_p f^2 \left(h_r \left(1-\frac{d}{r}\right) + h_s \left(1-\frac{d}{r}\right)\right)}{K_r(1-f)\left(h_r f + h_s(1-f)\left(1-\frac{d}{p}\right)\right)}}. \tag{16}$$

Due to the positive integer restriction, the following conditions must be satisfied:

$$R^{*(1,R)} = \left\lfloor \sqrt{\frac{K_p f^2 \left(h_r \left(1-\frac{d}{r}\right) + h_s \left(1-\frac{d}{r}\right)\right)}{K_r(1-f)\left(h_r f + h_s(1-f)\left(1-\frac{d}{p}\right)\right)}} \right\rfloor, \quad \text{when } TC^{(1,R)}(R^*) \le TC^{(1,R)}(R^*+1) \tag{17}$$

$$R^{*(1,R)} = \left\lfloor \sqrt{\frac{K_p f^2 \left(h_r \left(1-\frac{d}{r}\right) + h_s \left(1-\frac{d}{r}\right)\right)}{K_r(1-f)\left(h_r f + h_s(1-f)\left(1-\frac{d}{p}\right)\right)}} \right\rfloor + 1, \quad \text{when } TC^{(1,R)}(R^*) \le TC^{(1,R)}(R^*-1). \tag{18}$$

2.2. Model 2: $(P, 1)$ policy

In the $(P, 1)$ policy, there are P production setups and one rework per cycle. The following cost expression model of the total inventory cost is obtained from Eq. (7) of [1]:

$$TC^{(P,1)}(Q_p, Q_r) = \frac{K_p d(1-f)}{Q_p} + \frac{K_r df}{Q_r} + \frac{h_s}{2}\left((1-f)\left(1-\frac{d}{p}\right)Q_p + f\left(1-\frac{d}{r}\right)Q_r\right) + \frac{h_r}{2}\left(1-\frac{fd}{r}\right)Q_r. \tag{19}$$

From Eq. (8) of [1]:

$$Q_r(1-f) = PQ_p f. \tag{20}$$

After rearranging, one has:

$$Q_r = \frac{PQ_p f}{(1-f)}. \tag{21}$$

Teunter [1] found the optimal solution of Q_p and Q_r by differentiating (19) with respect to Q_p and Q_r and equating the result to zero, such that:

$$Q_p^{(P,1)} = \sqrt{\frac{2K_p d(1-f)}{h_s(1-f)\left(1-\frac{d}{p}\right)}} \quad \text{and} \quad Q_r^{(P,1)} = \sqrt{\frac{2K_r df}{h_s\left(1-\frac{d}{r}\right) + h_r\left(1-\frac{fd}{r}\right)}}. \tag{22}$$

Teunter modified the optimal P so that the variable was discrete. The modification formula for $(P, 1)$ model was:

$$\tilde{Q}_r^{(P,1)} = \frac{\tilde{P}^{(P,1)} Q_p^{(P,1)} f}{1-f} \tag{23}$$

where

$$\tilde{P}^{(P,1)} = \max\{1, [P^{(P,1)}]\} \tag{24}$$

was the positive integer nearest to $P^{(P,1)}$.

In our analysis, we substitute (21) into (19), resulting in:

$$\begin{aligned} TC^{(P,1)}(Q_p, Q_r) = {} & \frac{K_p d(1-f)}{Q_p} + \frac{K_r d(1-f)}{PQ_p} + \frac{h_s}{2}\left((1-f)\left(1-\frac{d}{p}\right)Q_p + f^2\left(1-\frac{d}{r}\right)\left(\frac{PQ_p}{(1-f)}\right)\right) \\ & + \frac{h_r}{2}\left(1-\frac{fd}{r}\right)\left(\frac{PQ_p f}{(1-f)}\right). \end{aligned} \tag{25}$$

Eq. (25) can be rewritten as:

$$TC^{(P,1)}(Q_p) = \frac{A_2}{Q_p} + B_2 Q_p \tag{26}$$

where:

$$A_2 = K_p d(1-f) + \frac{K_r d(1-f)}{P}$$

$$B_2 = \frac{h_s}{2}\left((1-f)\left(1-\frac{d}{p}\right) + f^2\left(1-\frac{d}{r}\right)\left(\frac{P}{(1-f)}\right)\right) + \frac{h_r}{2}\left(1-\frac{fd}{r}\right)\left(\frac{Pf}{(1-f)}\right).$$

Rewrite (26) using the method of perfect square format, one has:

$$TC^{(P,1)}(Q_p) = \frac{1}{Q_p}\left(\sqrt{A_2} - Q_p\sqrt{B_2}\right)^2 + 2\sqrt{A_2B_2}. \tag{27}$$

From (27), the minimum total cost occurs when $(\sqrt{A_2} - Q_p\sqrt{B_2}) = 0$. Therefore one has:

$$Q_p{}^{*(P,1)} = \sqrt{\frac{A_2}{B_2}} \tag{28}$$

and

$$TC^{(P,1)}(Q_p{}^*) = 2\sqrt{A_2B_2}. \tag{29}$$

Substitute A_2 and B_2 into (28), resulting in:

$$Q_p{}^{*(p,1)} = \sqrt{\frac{2d(1-f)\left(K_p + \frac{K_r}{P}\right)}{h_s\left((1-f)\left(1-\frac{d}{p}\right) + f^2\left(1-\frac{d}{r}\right)\left(\frac{P}{(1-f)}\right)\right) + h_r\left(1-\frac{fd}{r}\right)\left(\frac{pf}{(1-f)}\right)}}. \tag{30}$$

Eq. (29) can be rewritten as:

$$TC^{(P,1)}(Q_p{}^*) = 2\left(\frac{X_2}{R} + Y_2R + K_pd(1-f)\left(\frac{h_s}{2}(1-f)\left(1-\frac{d}{p}\right)\right) + K_rdf\left(\frac{h_r}{2}\left(1-\frac{fd}{r}\right) + \frac{h_s}{2}f\left(1-\frac{d}{r}\right)\right)\right)^{1/2} \tag{31}$$

where:

$$X_2 = \frac{K_rd(1-f)^2h_s}{2}\left(1-\frac{d}{p}\right)$$

and

$$Y_2 = K_pdf\left(\frac{h_r}{2}\left(1-\frac{fd}{r}\right) + \frac{h_s}{2}f\left(1-\frac{d}{r}\right)\right).$$

Rewriting (31) using the perfect square method format, one has:

$$\begin{aligned} TC^{(P,1)}(Q_p) = {} & 2\left(\frac{1}{P}\left(\sqrt{X_2} - P\sqrt{Y_2}\right)^2 + 2\sqrt{X_2Y_2} + K_pd(1-f)\left(\frac{h_s}{2}(1-f)\left(1-\frac{d}{p}\right)\right)\right. \\ & \left. + \; K_rdf\left(\frac{h_r}{2}\left(1-\frac{fd}{r}\right) + \frac{h_s}{2}f\left(1-\frac{d}{r}\right)\right)\right)^{1/2}. \end{aligned} \tag{32}$$

From (32), the total cost is minimum when $(\sqrt{X_2} - P\sqrt{Y_2}) = 0$. Therefore:

$$P^{*(P,1)} = \sqrt{\frac{X_2}{Y_2}}. \tag{33}$$

Substitute the original X_2 and Y_2 values into (33), such that:

$$P^{*(P,1)} = \sqrt{\frac{K_r(1-f)^2h_s\left(1-\frac{d}{p}\right)}{K_pf\left(h_r\left(1-\frac{fd}{r}\right) + h_sf\left(1-\frac{d}{r}\right)\right)}}. \tag{34}$$

Due to the positive integer restriction, the following conditions must be satisfied:

$$P^{*(P,1)} = \left\lfloor \sqrt{\frac{K_r(1-f)^2h_s\left(1-\frac{d}{p}\right)}{K_pf\left(h_r\left(1-\frac{fd}{r}\right) + h_sf\left(1-\frac{d}{r}\right)\right)}} \right\rfloor, \quad \text{when } TC^{(P,1)}(P^*) \le TC^{(P,1)}(P^*+1) \tag{35}$$

$$P^{*(P,1)} = \left\lfloor \sqrt{\frac{K_r(1-f)^2h_s\left(1-\frac{d}{p}\right)}{K_pf\left(h_r\left(1-\frac{fd}{r}\right) + h_sf\left(1-\frac{d}{r}\right)\right)}} \right\rfloor + 1, \quad \text{when } TC^{(P,1)}(P^*) \le TC^{(P,1)}(P^*-1). \tag{36}$$

Table 1
The optimal solutions of $(1, R)$ policy.

Problem	Teunter's model			Our study		TC difference (%)
	$(Q_p, Q_r, \tilde{R}^{(1,R)})$	$R^{(1,R)}$	$TC(Q_p, Q_r, \tilde{R}^{(1,R)})$ ($/unit time)	$(Q_p, Q_r, \tilde{R}^{(1,R)})$	$TC(Q_p, Q_r, \tilde{R}^{(1,R)})$ ($/unit time)	
Example	(53, 35.4, 6)	5.7	386.6	(51.75, 34.5, 6)	386.44	0.04
Case 1	(52.9, 79.3, 1)	0.87	433.4	(48.4, 72.6, 1)	431.6	0.42
Case 2	(37.2, 49.6, 3)	2.9	99.168	(3.68, 49.1, 3)	99.161	0.007
Case 3	(16.5, 29.7, 5)	4.6	165.7	(13.7, 30.8, 4)	145.2	12.4
Case 4	(29.9, 69.7, 1)	0.98	407.43	(29.7, 69.3, 1)	407.39	0.01
Case 5	(105.6, 158.4, 1)	1.22	4173.6	(117.6, 176.4, 2)	3673.4	12.0
Case 6	(98.5. 147.7, 1)	1.3	4459.7	(115.2, 86.4, 2)	3877	13.1
Case 7	(105.7, 123.3, 2)	1.78	3333.8	(100.9, 117.8, 2)	3331.1	0.08
Case 8	(70.9, 212.6, 3)	3.2	3631.8	(72.4, 217.2, 3)	3631.1	0.02
Case 9	(65.8, 197.3, 3)	2.58	1556.5	(47.9, 187.2, 3)	1554.2	0.15

Table 2
The optimal solutions of $(P, 1)$ policy.

Problem	Teunter's model			Our study		TC difference (%)
	$(Q_p, Q_r, \tilde{P}^{(P,1)})$	$P^{(P,1)}$	$TC(Q_p, Q_r, \tilde{P}^{(P,1)})$ ($/unit time)	$(Q_p, Q_r, \tilde{P}^{(P,1)})$	$TC(Q_p, Q_r, \tilde{P}^{(P,1)})$ ($/unit time)	
Example	(70.7, 282.8, 1)	0.1	1088.9	(18.63, 74.54, 1,)	536.7	50.7
Case 1	(85.2, 127.8, 1)	0.4	502.5	(48.4, 72.6, 1)	431.6	14.1
Case 2	(76.5, 306, 1)	0.15	244.3	(18.7, 74.8, 1)	112.8	53.8
Case 3	(54.6, 490.5, 1)	0.06	1086.5	(5.2, 46.8, 1)	206.8	81.0
Case 4	(62.2, 145.1, 1)	0.35	523.8	(29.7, 69.3, 1)	407.4	22.2
Case 5	(186.2, 279.3, 1)	0.46	4596.4	(117.6, 176.4, 1)	4150.5	9.7
Case 6	(191, 286.5, 1)	0.4	4980	(115.2, 172.8, 1)	4405.28	11.5
Case 7	(211.7, 494, 1)	0.2	6049.2	(66.5, 268.8, 1)	3458.9	42.8
Case 8	(197.6, 1778.4, 1)	0.11	12701.2	(33.2, 298.8, 1)	4148.9	67.3
Case 9	(234.6, 2111.4, 1)	0.08	6634.6	(30.8, 277.2,1)	1712.4	74.2

3. Verification with numerical example

A numerical example is provided to compare the optimal solution of our study with that of [1]. We use the same example as [1] and generate data for 9 cases. The data are shown in the Appendix. Table 1 compares the optimal solutions of Teunter's with our study for $(1, R)$ policy and Table 2 compares the optimal solutions of Teunter's with our study for $(P, 1)$ policy.

The percentage total cost difference between Teunter's model and our study is calculated as follows:

$$\%\text{ Total cost difference} = \left(\frac{TC_{\text{Teunter's}} - TC_{\text{our study}}}{TC_{\text{Teunter's}}}\right) \times 100\%.$$

Tables 1 and 2 show that our studies always have a lower total cost than Teunter's. Table 1 shows the total cost from our study in $(1, R)$ policy are 0.007% to 13.1% lower, and Table 2 shows the total cost from our study in $(P, 1)$ policy are 9.7% to 81 % lower. The total cost percentage differences between Teunter's and our model for $(P, 1)$ policy tends to bigger than $(1, R)$ policy. This is because the deviation of the optimal $P(P^{(P,1)})$ for the discrete value of $P(\tilde{P}^{(P,1)})$ is bigger than the deviation of the optimal $R(R^{(1,R)})$ for the discrete value of $R(\tilde{R}^{(1,R)})$.

4. Conclusion

This note has explored the inventory models with product recovery from [1]. The main purpose of this note is to suggest an integrated solution based on algebraic approach for determining the number of production and recovery lot sizes with product recovery. The study not only provides an easy to follow approach to derive the optimal solution, it also results in a lower total cost as seen in Tables 1 and 2.

One limitation of this study is the assumption that all return items are redeemable or as-good-as-new, and the demand rate and return fraction are deterministic. Future work can be done to consider stochastic demand rate and return fraction, as well as deleting the assumption that all return items can be redeemed.

Appendix

	Parameters							
	D	f	p	r	K_p	K_r	h_r	h_s
Example	1000	0.8	5 000	3 000	20	5	2	10
Case 1	871	0.6	144	1 068	18	12	7	11
Case 2	480	0.8	1 129	1 001	7	4	1	2
Case 3	285	0.9	1 221	1 152	12	7	3	3
Case 4	877	0.7	1 927	1 125	12	11	8	10
Case 5	9867	0.6	26 987	15 879	40	28	15	31
Case 6	9330	0.6	29 844	16 551	43	25	17	32
Case 7	7373	0.7	11 966	17 655	28	24	16	24
Case 8	8704	0.9	25 669	13 152	43	36	12	29
Case 9	3255	0.9	5 172	4 213	47	34	10	15

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